# CaMPL Type Inference

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# Reimplementing CaMPL

- CaMPL is a Concurrent functional style programming language.
- In the current implementation of CaMPL, functions are not first-class citizens. Higher-order functions in the current implementation can be added using the coinductive data type:

```
codata S -> App(A,B)  =
    App ::  A,S -> B
```

However, this results in a cumbersome syntax.

- Folds for data, Unfolds for codata and Drives for protocols have not been implemented.

# CaMPL Type Inference

- CaMPL is a statically typed programming language. The compiler must check that each expression can be typed before the program runs.
- CaMPL uses a type inference algorithm, allowing types to be inferred from the context of the expression.

```
addition = x, y -> (((+) x) y)
```

```
addition :: (Integer, Integer) -> Integer
addition = x, y -> (((+) x) y)
```

```haskell
f (g, h) = g (h 0)
```

$$h :: Int \to a$$

$$g :: a \to b$$

$$\Longrightarrow$$

$$f :: (a \to b, Int \to a) \to b$$

We can work out the type of f in this way, but we need an algorithm to be able to infer the types of arbitrary expressions.

# Type Inference Algorithm

This algorithm has two parts:

1. Collecting the equations which must hold between the types.
2. Solving these equations.

To do Type Inference for CaMPL, one needs type inference rules for all its constructs.

To obtain the type Inference rules, we transform type checking judgements into type inference judgements:

$$\Gamma \vdash t :: T \longrightarrow \Gamma \vdash t :: T\langle E \rangle$$

# Type Checking Rules for the Simply Typed Lambda Calculus

1. Choose a variable from the context

$$\boxed{\frac{}{x :: P, \Gamma \vdash x :: P}\ \mathrm{prj}}$$

2. Form an abstraction

$$\boxed{\frac{x :: P, \Gamma \vdash t :: T}{\Gamma \vdash \lambda x.t :: P \to T}\ \mathrm{abst}}$$

3. Form an application

$$\boxed{\frac{\Gamma \vdash f :: P \to Q \qquad \Gamma \vdash t :: P}{\Gamma \vdash (ft) :: Q}\ \mathrm{app}}$$

# Type Inference Rules for the Simply Typed Lambda Calculus

1. Type Infer Variables

$$\frac{}{x :: Q, \Gamma \vdash\ x\ ::\ Q}\ \text{prj}$$

$$\boxed{\frac{}{x :: P, \Gamma \vdash\ x\ ::\ Q \langle\, P = Q \,\rangle}\ \text{prj}}$$

2. Type Infer Abstractions

$$\frac{x :: P, \Gamma \vdash\ t\ ::\ T}{\Gamma \vdash\ \lambda x.t\ ::\ P \to T}\ \text{abst}$$

$$\boxed{\frac{x :: P, \Gamma \vdash t :: T \langle\, E_1 \,\rangle}{\Gamma \vdash\ \lambda x.t ::\ Q \quad \langle\, \exists\, P, T.Q\ = P \to T, E_1 \,\rangle}\ \text{abst}}$$

3. Type Infer Applications

$$\frac{\Gamma \vdash\ f :: P \to Q \qquad \Gamma \vdash\ t :: P}{\Gamma \vdash\ (ft)\ ::\ Q}\ \text{app}$$

$$\boxed{\frac{\Gamma \vdash\ f ::\ F \langle E_1 \rangle \qquad \Gamma \vdash t ::\ P \langle E_2 \rangle}{\Gamma \vdash\ ft ::\ Q \quad \langle\, \exists\, F, P.F\ =\ P \to Q, E_1, E_2 \,\rangle}\ \text{app}}$$

# Type Inference Example

$$\frac{x :: P, \Gamma \ \vdash \ t :: \ T \ \langle \ E_1 \ \rangle}{\Gamma \ \vdash \ \lambda x.t :: \ Q \quad \langle \ \exists \, P, T.Q \ = P \rightarrow T, E_1 \ \rangle} \text{abst}$$

$$\frac{\begin{array}{c} x :: P \vdash x :: E \\ \langle E_1 \rangle \end{array}}{\begin{array}{c} \vdash \lambda x.x :: Q \\ E_0 = \langle \exists \, P, E.Q = P \rightarrow E, E_1 \rangle \end{array}} \text{abst}$$

# Type Inference Example

$$\dfrac{}{x :: P, \Gamma \vdash x :: Q \,\langle\, P = Q \,\rangle}\ \text{prj}$$

$$\dfrac{\dfrac{}{\begin{array}{c} x :: P \vdash x :: E \\ E_1 = \langle P = E \rangle \end{array}}\text{prj}}{\begin{array}{c} \vdash \lambda x.x :: Q \\ E_0 = \langle \exists\, P, E. Q = P \to E, E_1 \rangle \end{array}}\text{abst}$$

# Type Equations

$$\dfrac{\dfrac{}{x :: P \vdash x :: E}\,\text{prj} \qquad E_1 = \langle P = E\rangle}{\vdash \lambda x.x :: Q \qquad E_0 = \langle \exists\, P, E.Q = P \to E, E_1\rangle}\,\text{abst}$$

$$\langle \exists\, P, E.\, Q = P \to E \\ \langle P = E\rangle\rangle$$

# Solving Type Equations

1. Existentially bound variables can be eliminated if there is an occurs check free substitution for them

$$\exists\, x.(x = t, E) = E[t/x]$$

$$\begin{array}{r} \langle \exists\, A, B.\, C = (A, B) \\ \langle A = D \rangle \\ \langle B = E \rangle\rangle \end{array} \quad \Rightarrow \quad \langle C = (D, E)\rangle$$

2. $f(t_1, \ldots, t_n) = f(t'_1, \ldots, t'_n)$ can be replaced with $t_1 = t'_1, \ldots, t_n = t'_n$ (matching).

$$\langle A \to B \to C = D \to E \rangle \quad \Rightarrow \quad \begin{array}{c} \langle B \to C = E \rangle \\ \langle A = D \rangle \end{array}$$

# Type Equations

$$\langle \exists \cancel{P}, E.\, \cancel{Q = P \to E} \qquad \langle Q = E \to E \rangle$$

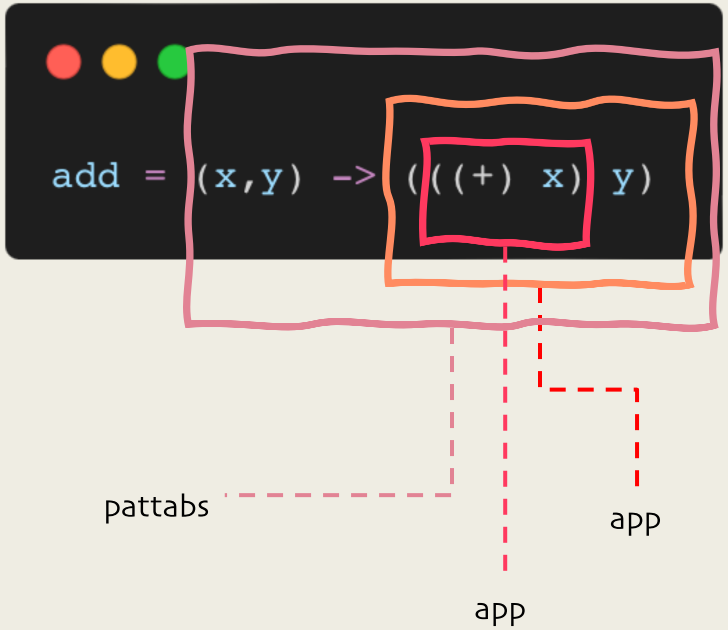
$$\cancel{\langle P = E \rangle}\rangle$$

# CaMPL Constructs

Type judgements can be written for every construct in CaMPL.

```
add = (x,y) -> (((+) x) y)
```

# CaMPL Constructs



# Addition Function Type Inference

$$\frac{\text{funname} :: \text{Q}, \Gamma \vdash \begin{array}{c}\text{patt}_1 \rightarrow \text{exp}_1 \\ \vdots \\ \text{patt}_n \rightarrow \text{exp}_n\end{array} :: \text{Q} \quad \langle E_0 \rangle}{\Gamma \vdash \begin{array}{c}\text{funname} = \text{patt}_1 \rightarrow \text{exp}_1 \\ \vdots \\ \text{patt}_n \rightarrow \text{exp}_n\end{array} :: \text{Q} \quad \langle E_0 \rangle}\ \text{function}$$

$$\frac{\begin{array}{c} add :: 0 \vdash (x, y) \rightarrow (((+)\ x)\ y) :: 0 \\ \langle E_1 \rangle \end{array}}{\begin{array}{c} \vdash add = (x, y) \rightarrow (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle \end{array}}\ \text{function}$$

# Addition Function Type Inference

$$\frac{x :: P, \Gamma \vdash t :: T \langle E_1 \rangle}{\Gamma \vdash x \to t :: Q \quad \langle \exists P, T.Q = P \to T, E_1 \rangle} \text{pattabs}$$

$$\frac{\boxed{\dfrac{add :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \quad \langle E_2 \rangle}{add :: 0 \vdash (x, y) \to (((+)\ x)\ y) :: 0 \quad E_1 = \langle \exists 1, 2.0 = 1 \to 2, E_2 \rangle} \text{pattabs}}}{\vdash add = (x, y) \to (((+)\ x)\ y) \quad E_0 = \langle E_1 \rangle} \text{function}$$

# Addition Function Type Inference

$$\frac{\Gamma \vdash x :: A \langle E_1 \rangle \qquad \Gamma \vdash y :: B \langle E_2 \rangle}{\begin{array}{c}\Gamma \vdash (x, y) :: Q \\ \langle \exists A, B.Q = (A, B), E_1, E_2 \rangle\end{array}} \text{tuple}$$

$$\cfrac{\cfrac{\boxed{\cfrac{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (((+)\ x)\ y) :: 2 \\ \langle E_3 \rangle \end{array}}{\begin{array}{c} add :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \\ E_2 = \langle \exists\, 3, 4.1 = (3, 4), E_3 \rangle \end{array}} \text{tuple}}}{\begin{array}{c} add :: 0 \vdash (x, y) \to (((+)\ x)\ y) :: 0 \\ E_1 = \langle \exists\, 1, 2.0 = 1 \to 2, E_2 \rangle \end{array}} \text{pattabs}}{\begin{array}{c} \vdash add = (x, y) \to (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle \end{array}} \text{function}$$

# Addition Function Type Inference

$$\frac{\Gamma \vdash f :: F \, \langle E_1 \rangle \quad \Gamma \vdash t :: P \, \langle E_2 \rangle}{\begin{array}{c}\Gamma \vdash ft :: Q \\ \langle \exists F, P.F = P \to Q, E_1, E_2 \rangle\end{array}} \text{app}$$

$$\boxed{\frac{\begin{array}{c}add :: 0, x :: 3, y :: 4 \vdash ((+)\ x) :: 5 \\ \langle E_4 \rangle\end{array} \quad \begin{array}{c}add :: 0, x :: 3, y :: 4 \vdash y :: 6 \\ \langle E_5 \rangle\end{array}}{\begin{array}{c}add :: 0, x :: 3, y :: 4 \vdash (((+)\ x)\ y) :: 2 \\ E_3 = \langle \exists\, 5, 6.5 = 6 \to 2, E_4, E_5 \rangle\end{array}} \text{app}}$$

$$\frac{}{\begin{array}{c}add :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \\ E_2 = \langle \exists\, 3, 4.1 = (3, 4), E_3 \rangle\end{array}} \text{tuple}$$

$$\frac{}{\begin{array}{c}add :: 0 \vdash (x, y) \to (((+)\ x)\ y) :: 0 \\ E_1 = \langle \exists\, 1, 2.0 = 1 \to 2, E_2 \rangle\end{array}} \text{pattabs}$$

$$\frac{}{\begin{array}{c}\vdash add = (x, y) \to (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle\end{array}} \text{function}$$

# Addition Function Type Inference

$$\frac{}{\begin{array}{c} x :: P, \Gamma \vdash x :: Q \\ \langle P = Q \rangle \end{array}} \text{prj}$$

$$\dfrac{\dfrac{\dfrac{\begin{array}{c} \mathit{add} :: 0, x :: 3, y :: 4 \vdash ((+)\ x) :: 5 \\ \langle E_4 \rangle \end{array} \quad \dfrac{}{\begin{array}{c} \mathit{add} :: 0, x :: 3, y :: 4 \vdash y :: 6 \\ E_5 = \langle 4 = 6 \rangle \end{array}} \text{prj}}{\begin{array}{c} \mathit{add} :: 0, x :: 3, y :: 4 \vdash (((+)\ x)\ y) :: 2 \\ E_3 = \langle \exists\, 5, 6.5 = 6 \rightarrow 2, E_4, E_5 \rangle \end{array}} \text{app}}{\begin{array}{c} \mathit{add} :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \\ E_2 = \langle \exists\, 3, 4.1 = (3, 4), E_3 \rangle \end{array}} \text{tuple}}{\dfrac{\begin{array}{c} \mathit{add} :: 0 \vdash (x, y) \rightarrow (((+)\ x)\ y) :: 0 \\ E_1 = \langle \exists\, 1, 2.0 = 1 \rightarrow 2, E_2 \rangle \end{array}}{\begin{array}{c} \vdash \mathit{add} = (x, y) \rightarrow (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle \end{array}} \text{function}} \text{pattabs}$$

# Addition Function Type Inference

$$\dfrac{\Gamma \vdash f :: F\ \langle E_1 \rangle \qquad \Gamma \vdash t :: P\ \langle E_2 \rangle}{\begin{array}{c}\Gamma \vdash ft :: Q \\ \langle\, \exists F, P.F = P \to Q, E_1, E_2 \,\rangle\end{array}}\ \text{app}$$

$$\dfrac{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (+) :: 7 \\ \langle E_6 \rangle \end{array} \qquad \begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash x :: 8 \\ \langle E_7 \rangle \end{array}}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash ((+)\ x) :: 5 \\ E_4 = \langle \exists\, 7, 8.7 = 8 \to 5, E_6, E_7 \rangle \end{array}}\ \text{app}$$

$$\dfrac{}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash y :: 6 \\ E_5 = \langle 4 = 6 \rangle \end{array}}\ \text{prj}$$

$$\dfrac{\cdots}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (((+)\ x)\ y) :: 2 \\ E_3 = \langle \exists\, 5, 6.5 = 6 \to 2, E_4, E_5 \rangle \end{array}}\ \text{app}$$

$$\dfrac{}{\begin{array}{c} add :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \\ E_2 = \langle \exists\, 3, 4.1 = (3, 4), E_3 \rangle \end{array}}\ \text{tuple}$$

$$\dfrac{}{\begin{array}{c} add :: 0 \vdash (x, y) \to (((+)\ x)\ y) :: 0 \\ E_1 = \langle \exists\, 1, 2.0 = 1 \to 2, E_2 \rangle \end{array}}\ \text{pattabs}$$

$$\dfrac{}{\begin{array}{c} \vdash add = (x, y) \to (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle \end{array}}\ \text{function}$$

# Addition Function Type Inference

$$\frac{}{\begin{array}{c} x :: P, \Gamma \vdash x :: Q \\ \langle P = Q \rangle \end{array}}\,\text{prj}$$

$$\dfrac{\dfrac{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (+) :: 7 \\ E_6 = \langle 7 = Int \to Int \to Int \rangle \end{array} \quad \boxed{\dfrac{}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash x :: 8 \\ E_7 = \langle 3 = 8 \rangle \end{array}}\,\text{prj}}}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash ((+)\ x) :: 5 \\ E_4 = \langle \exists\, 7, 8.7 = 8 \to 5, E_6, E_7 \rangle \end{array}}\,\text{app} \quad \dfrac{}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash y :: 6 \\ E_5 = \langle 4 = 6 \rangle \end{array}}\,\text{prj}}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (((+)\ x)\ y) :: 2 \\ E_3 = \langle \exists\, 5, 6.5 = 6 \to 2, E_4, E_5 \rangle \end{array}}\,\text{app}$$

$$\dfrac{\dfrac{\dfrac{\cdots}{\begin{array}{c} add :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \\ E_2 = \langle \exists\, 3, 4.1 = (3, 4), E_3 \rangle \end{array}}\,\text{tuple}}{\begin{array}{c} add :: 0 \vdash (x, y) \to (((+)\ x)\ y) :: 0 \\ E_1 = \langle \exists\, 1, 2.0 = 1 \to 2, E_2 \rangle \end{array}}\,\text{pattabs}}{\begin{array}{c} \vdash add = (x, y) \to (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle \end{array}}\,\text{function}$$

# Type Equations

$$
\begin{array}{l}
\langle \exists\, 1, 2.\ 0 = 1 \to 2 \\
\quad \langle \exists\, 3, 4.\ 1 = (3, 4) \\
\quad\quad \langle \exists\, 5, 6.\ 5 = 6 \to 2 \\
\quad\quad\quad \langle \exists\, 7, 8.\ 7 = 8 \to 5 \\
\quad\quad\quad\quad \langle 7 = Int \to Int \to Int \rangle \\
\quad\quad\quad\quad \langle 3 = 8 \rangle\rangle \\
\quad\quad\quad \langle 4 = 6 \rangle\rangle\rangle
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (+) :: 7 \\ E_6 = \langle 7 = Int \to Int \to Int \rangle \end{array} \qquad \dfrac{}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash x :: 8 \\ E_7 = \langle 3 = 8 \rangle \end{array}}\,\text{prj}}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash ((+)\ x) :: 5 \\ E_4 = \langle \exists\, 7, 8.7 = 8 \to 5, E_6, E_7 \rangle \end{array}}\,\text{app} \qquad \dfrac{}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash y :: 6 \\ E_5 = \langle 4 = 6 \rangle \end{array}}\,\text{prj}}{\begin{array}{c} add :: 0, x :: 3, y :: 4 \vdash (((+)\ x)\ y) :: 2 \\ E_3 = \langle \exists\, 5, 6.5 = 6 \to 2, E_4, E_5 \rangle \end{array}}\,\text{app}}{\begin{array}{c} add :: 0, (x, y) :: 1 \vdash (((+)\ x)\ y) :: 2 \\ E_2 = \langle \exists\, 3, 4.1 = (3, 4), E_3 \rangle \end{array}}\,\text{tuple}}{\begin{array}{c} add :: 0 \vdash (x, y) \to (((+)\ x)\ y) :: 0 \\ E_1 = \langle \exists\, 1, 2.0 = 1 \to 2, E_2 \rangle \end{array}}\,\text{pattabs}}{\begin{array}{c} \vdash add = (x, y) \to (((+)\ x)\ y) \\ E_0 = \langle E_1 \rangle \end{array}}\,\text{function}
$$

$$
\begin{array}{l}
\langle \exists\, 1, 2.\ 0 = 1 \to 2 \\
\qquad \langle \exists\, 3, 4.\ 1 = (3, 4) \\
\qquad\qquad \langle \exists\, 5, 6.\ 5 = 6 \to 2 \\
\qquad\qquad\quad \boxed{\begin{array}{l}
\langle \exists\, 7, 8.\ 7 = 8 \to 5 \\
\qquad\quad \langle 7 = Int \to Int \to Int \rangle \\
\qquad\quad \langle 3 = 8 \rangle \rangle
\end{array}} \\
\qquad\qquad\quad \langle 4 = 6 \rangle \rangle \rangle
\end{array}
$$

$$\langle \exists\, 1, 2.\ 0 = 1 \to 2$$

$$\qquad \langle \exists\, 3, 4.\ 1 = (3, 4)$$

$$\qquad\qquad \langle \exists\, 5, 6.\ 5 = 6 \to 2 \qquad\qquad \langle 7 = 3 \to 5\rangle$$

$$\boxed{\begin{array}{l} \langle \exists\, 7, \cancel{8}.\ \cancel{7 = 8 \to 5} \\ \qquad \langle 7 = Int \to Int \to Int\rangle \\ \qquad \langle \cancel{3 = 8}\rangle\rangle \end{array}}$$

$$\qquad \langle 4 = 6\rangle\rangle\rangle$$

$$\langle \exists\, 1, 2.\ 0 = 1 \to 2$$

$$\langle \exists\, 3, 4.\ 1 = (3, 4)$$

$$\langle \exists\, 5, 6.\ 5 = 6 \to 2$$

$$\boxed{\langle \exists\, 7.\ 7 = 3 \to 5, 7 = Int \to Int \to Int, 4 = 6 \rangle}$$

$\langle 5 = Int \to Int \rangle$
$\langle 3 = Int \rangle$

matching

$\langle Int \to Int \to Int = 3 \to 5 \rangle$

$\langle \exists\, 1, 2.\ 0 = 1 \to 2$
$\langle \exists\, 3, 4.\ 1 = (3, 4)$
$\langle \exists\, 5, 6.\ 5 = 6 \to 2$
$\boxed{\langle \cancel{\exists\, 7.}\ \cancel{7 = 3 \to 5}, \cancel{7 = Int \to Int \to Int}, 4 = 6 \rangle}$

$$\langle \exists\, 1, 2.\ 0 = 1 \to 2$$

$$\langle \exists\, 3, 4.\ 1 = (3, 4)$$

$$\boxed{\langle \exists\, 5, 6.\ 5 = 6 \to 2, 3 = Int, 5 = Int \to Int, 4 = 6\rangle\rangle\rangle}$$

$\langle 6 = Int \rangle$
$\langle 2 = Int \rangle$

matching

$\langle Int \to Int = 6 \to 2 \rangle$

$$\langle \exists\, 1, 2.\ 0 = 1 \to 2$$
$$\langle \exists\, 3, 4.\ 1 = (3, 4)$$
$$\langle \cancel{\exists\, 5}, 6.\ \cancel{5 = 6 \to 2}, 3 = Int, 5 \cancel{= Int \to} Int, 4 = 6 \rangle\rangle\rangle$$

$$\begin{aligned}&\langle \exists\, 1, 2.\ 0 = 1 \to 2\\ &\qquad \langle \exists\, 3, 4.\ 1 = (3, 4)\\ &\qquad\quad \boxed{\langle \exists\, 6.\ 3 = Int, 2 = Int, 6 = Int, 4 = 6\rangle\rangle\rangle}\end{aligned}$$

$$
\begin{aligned}
&\langle \exists\, 1, 2.\ 0 = 1 \to 2 \\
&\qquad \langle \exists\, 3, 4.\ 1 = (3, 4) \qquad\qquad \color{red}{\langle 4 = Int \rangle} \\
&\qquad\quad \langle \cancel{\exists\, 6}.\ 3 = Int, 2 = Int, 6 \cancel{= Int}, 4 \cancel{= 6} \rangle \rangle \rangle
\end{aligned}
$$

$$
\begin{aligned}
&\langle \exists\, 1, 2.\ 0 = 1 \to 2 \\
&\qquad \boxed{\begin{aligned}&\langle \exists\, 3, 4.\ 1 = (3, 4) \\ &\qquad\qquad \langle\, 3 = Int, 2 = Int, 4 = Int \rangle\rangle\rangle\end{aligned}}
\end{aligned}
$$

$$
\begin{aligned}
&\langle \exists\, 1, 2.\; 0 = 1 \to 2 \\
&\quad \boxed{\begin{aligned}&\langle \exists\, \cancel{3, 4}.\; \cancel{1 = (3, 4)} \\ &\qquad \langle\, \cancel{3 = Int}, 2 = Int, \cancel{4 = Int} \rangle\rangle\rangle\end{aligned}} \qquad \langle 1 = (Int, Int) \rangle
\end{aligned}
$$

$$\boxed{\langle \exists\, 1, 2.\ 0 = 1 \to 2, 1 = (Int, Int), 2 = Int \rangle}$$

$$\boxed{\langle \cancel{\exists 1, 2.}\ 0 \cancel{= 1 \rightarrow 2}, \cancel{1 = (Int, Int), 2 = Int} \rangle}$$

$$\langle 0 = (Int, Int) \rightarrow Int \rangle$$

# CaMPL Inductive Constructs

```
data List(A) -> C =
    Nil :: -> C
    Cons :: (A,C) -> C


append = (Nil, ys) -> ys
         ( Cons(x, xs ), ys) -> Cons (x, append (xs, ys))
```

consPatt

cons

app

# CaMPL Data Type Rules

$$
\begin{array}{llcl}
data \quad List(A) & \to & C & = \\
\quad Nil :: & & \to & C \\
\quad Cons :: A, C & & \to & C
\end{array}
$$

$$
\dfrac{\Gamma \vdash f :: F \langle E_1 \rangle \quad \Gamma \vdash t :: P \langle E_2 \rangle}{\begin{array}{c}\Gamma \vdash ft :: Q \\ \langle \exists F, P.F = P \to Q, E_1, E_2 \rangle\end{array}} \text{app}
$$

$$
\dfrac{\begin{array}{c}\vdash Cons :: 1 \\ \langle E_1 \rangle\end{array} \qquad \begin{array}{c}\vdash (x, ys) :: 2 \\ \langle E_2 \rangle\end{array}}{\begin{array}{c}\vdash (Cons(x, ys)) :: 0 \\ \langle \exists 1, 2.1 = 2 \to 0, E_1, E_2 \rangle\end{array}} \text{app}
$$

# CaMPL Data Type Rules

$$\begin{array}{lllll} data & List(A) & \to & C & = \\ & \quad Nil :: & & \to & C \\ & \quad Cons :: A, C & & \to & C \end{array} \xrightarrow{[3/A,\ List(3)/C]} \begin{array}{lll} Nil :: & \to & List(3) \\ Cons :: 3, List(3) & \to & List(3) \end{array}$$

$$\dfrac{\boxed{\dfrac{}{\begin{array}{c}\vdash Cons :: 1 \\ E_1 = \langle \exists\, 3.1 = (3, List(3)) \to List(3)\rangle\end{array}}\text{cons}} \qquad \dfrac{}{\begin{array}{c}\vdash (x, ys) :: 2 \\ \langle E_2 \rangle\end{array}}}{\begin{array}{c}\vdash (Cons(x, ys)) :: 0 \\ \langle \exists\, 1, 2.1 = 2 \to 0, E_1, E_2 \rangle\end{array}}\text{app}$$

# CaMPL Data Type Rules

$$\begin{array}{llll} \mathit{data} & \mathit{List}(A) & \to & C \quad = \\ & \mathit{Nil} :: & \to & C \\ & \mathit{Cons} :: A, C & \to & C \end{array} \xrightarrow{[1/A,\ \mathit{List}(1)/C]} \begin{array}{lll} \mathit{Nil} :: & \to & \mathit{List}(1) \\ \mathit{Cons} :: 1, \mathit{List}(1) & \to & \mathit{List}(1) \end{array}$$

$$\boxed{\dfrac{\begin{array}{c}(x, xs) :: 2 \vdash \\ \langle E_1 \rangle\end{array}}{\begin{array}{c}(\mathit{Cons}(x, xs)) :: 0 \vdash \\ \langle \exists\, 1, 2.2 = (1, \mathit{List}(1)), 0 = \mathit{List}(1), E_1 \rangle\end{array}}\ \text{consPatt}}$$

```
errppend = (Nil, ys) -> ys
           (Cons (x, xs), ys) -> Cons(x, x)
```

$$\dfrac{}{\begin{array}{c} errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17 \\ E_{13} = \langle 12 = 17 \rangle \end{array}}\,\text{prj} \qquad \dfrac{}{\begin{array}{c} errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18 \\ E_{14} = \langle 12 = 18 \rangle \end{array}}\,\text{prj}$$

$$\dfrac{}{\begin{array}{c} errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14 \\ E_{11} = \langle \exists\, 16.14 = (16, List(16)) \to List(16) \rangle \end{array}}\,\text{cons} \qquad \dfrac{}{\begin{array}{c} errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15 \\ E_{12} = \langle \exists\, 17, 18.15 = (17, 18), E_{13}, E_{14} \rangle \end{array}}\,\text{tuple}$$

$$\dfrac{}{\begin{array}{c} errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x, x)) :: 4 \\ E_{10} = \langle \exists\, 14, 15.14 = 15 \to 4, E_{11}, E_{12} \rangle \end{array}}\,\text{app}$$

$$\dfrac{}{\begin{array}{c} errppend :: 0, ys :: 6 \vdash ys :: 2 \\ E_6 = \langle 2 = 6 \rangle \end{array}}\,\text{prj} \qquad \dfrac{}{\begin{array}{c} errppend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x, x)) :: 4 \\ E_9 = \langle \exists\, 12, 13.11 = (12, 13), E_{10} \rangle \end{array}}\,\text{tuple}$$

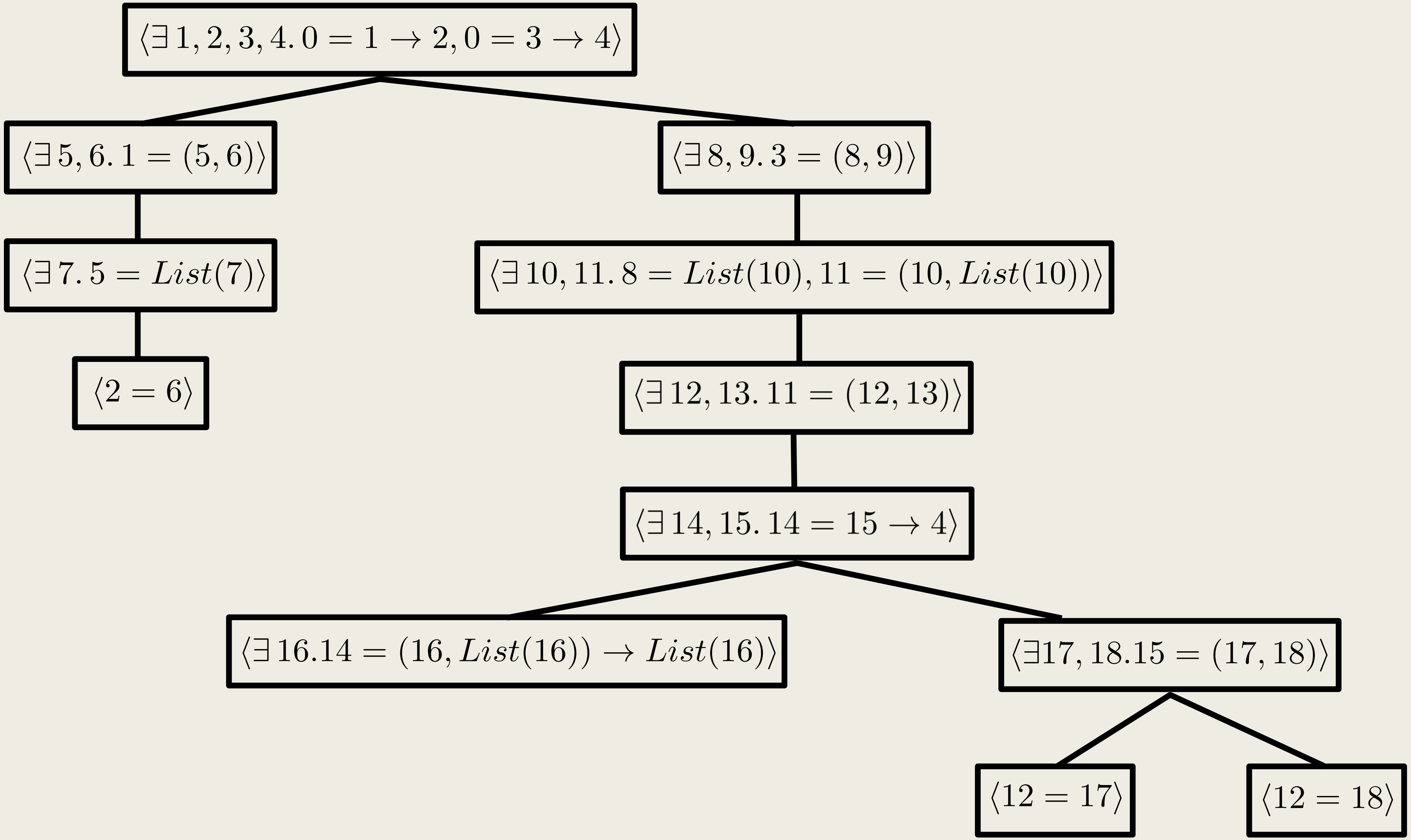
$$\dfrac{}{\begin{array}{c} errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2 \\ E_4 = \langle \exists\, 7.5 = List(7), E_5 \rangle \end{array}}\,\text{consPatt} \qquad \dfrac{}{\begin{array}{c} errppend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x, x)) :: 4 \\ E_8 = \langle \exists\, 10, 11.8 = List(10), 11 = (10, List(10)), E_9 \rangle \end{array}}\,\text{consPatt}$$

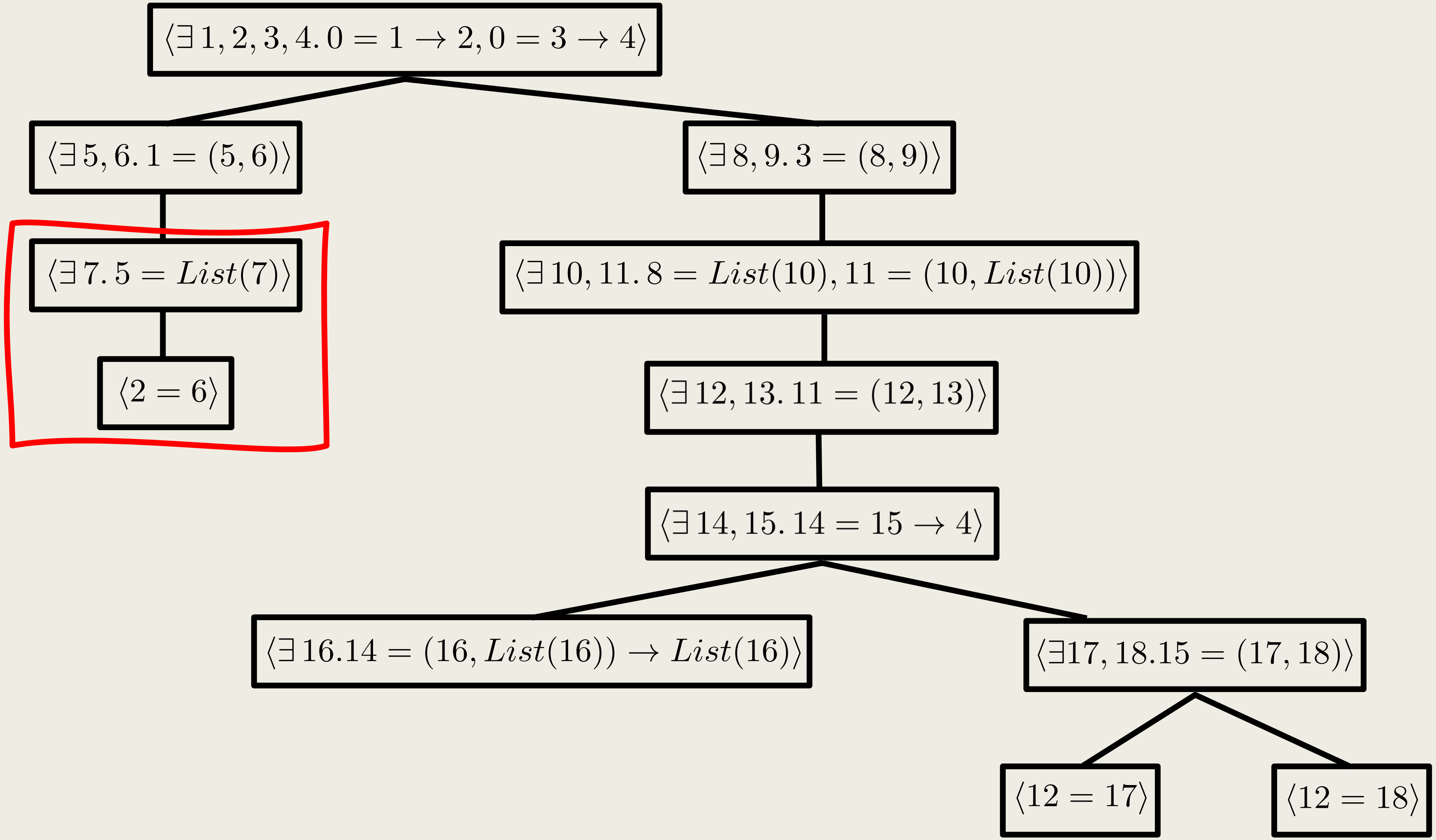
$$\dfrac{}{\begin{array}{c} errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2 \\ E_3 = \langle \exists\, 5, 6.1 = (5, 6), E_4 \rangle \end{array}}\,\text{tuple} \qquad \dfrac{}{\begin{array}{c} errppend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x, x)) :: 4 \\ E_7 = \langle \exists\, 8, 9.3 = (8, 9), E_8 \rangle \end{array}}\,\text{tuple}$$

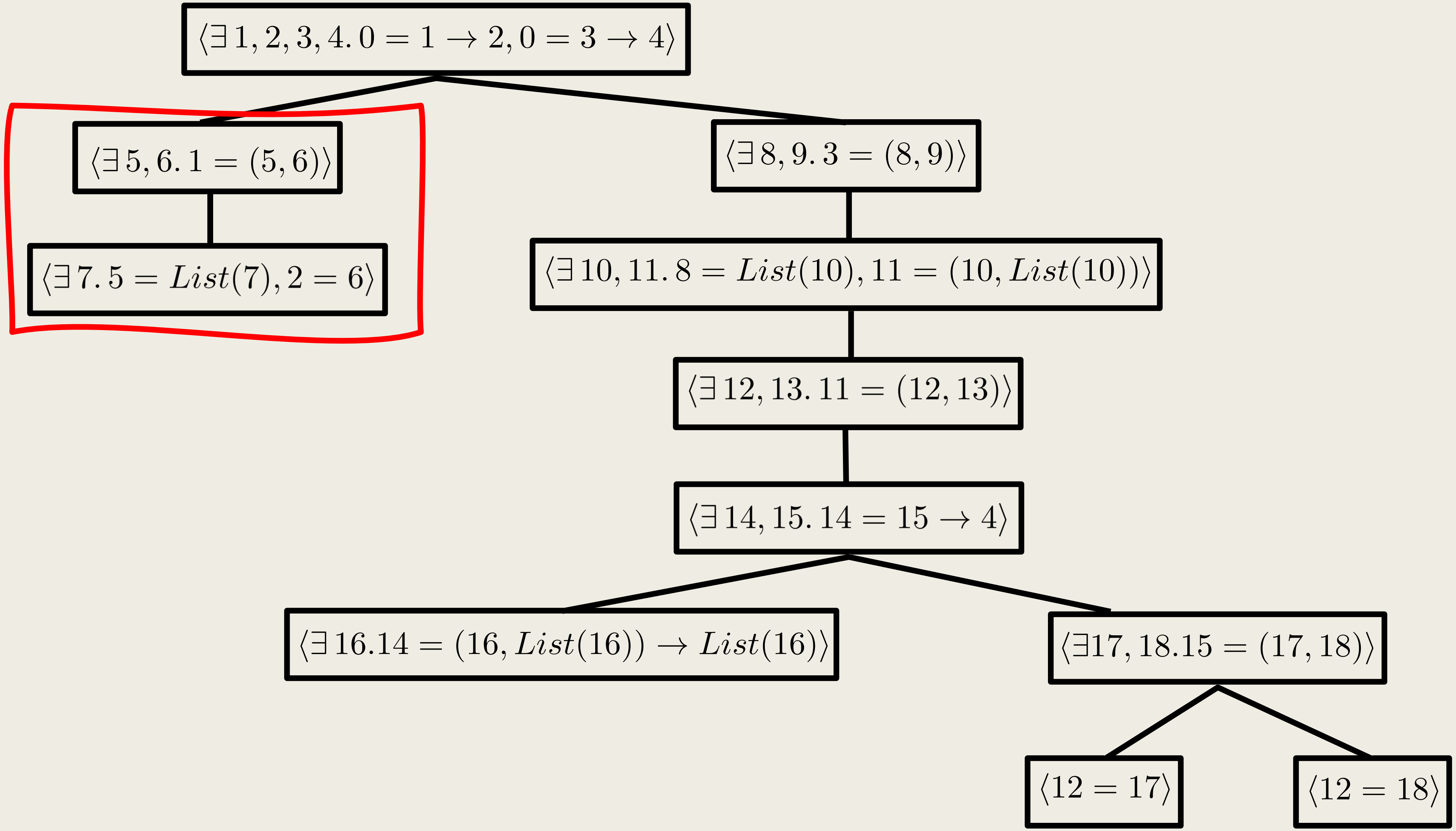
$$\dfrac{}{\begin{array}{ll} errppend :: 0 \vdash & \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0 \\ & E_1 = \langle \exists\, 1, 2, 3, 4.0 = 1 \to 2, 0 = 3 \to 4, E_3, E_7 \rangle \end{array}}\,\text{pattabs}$$

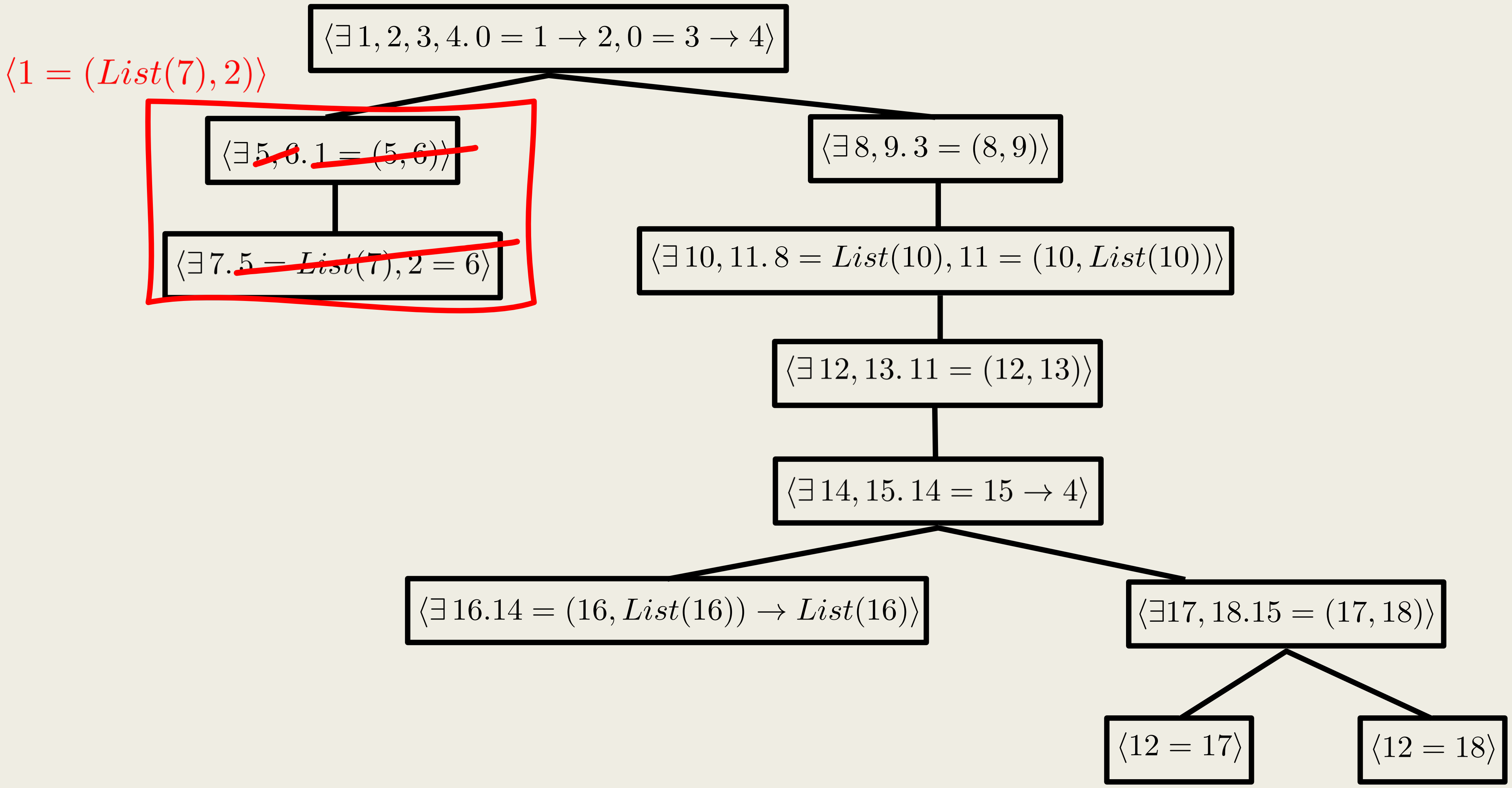
$$\dfrac{}{\begin{array}{ll} \vdash errppend = & \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} \\ & E_0 = \langle E_1 \rangle \end{array}}\,\text{function}$$

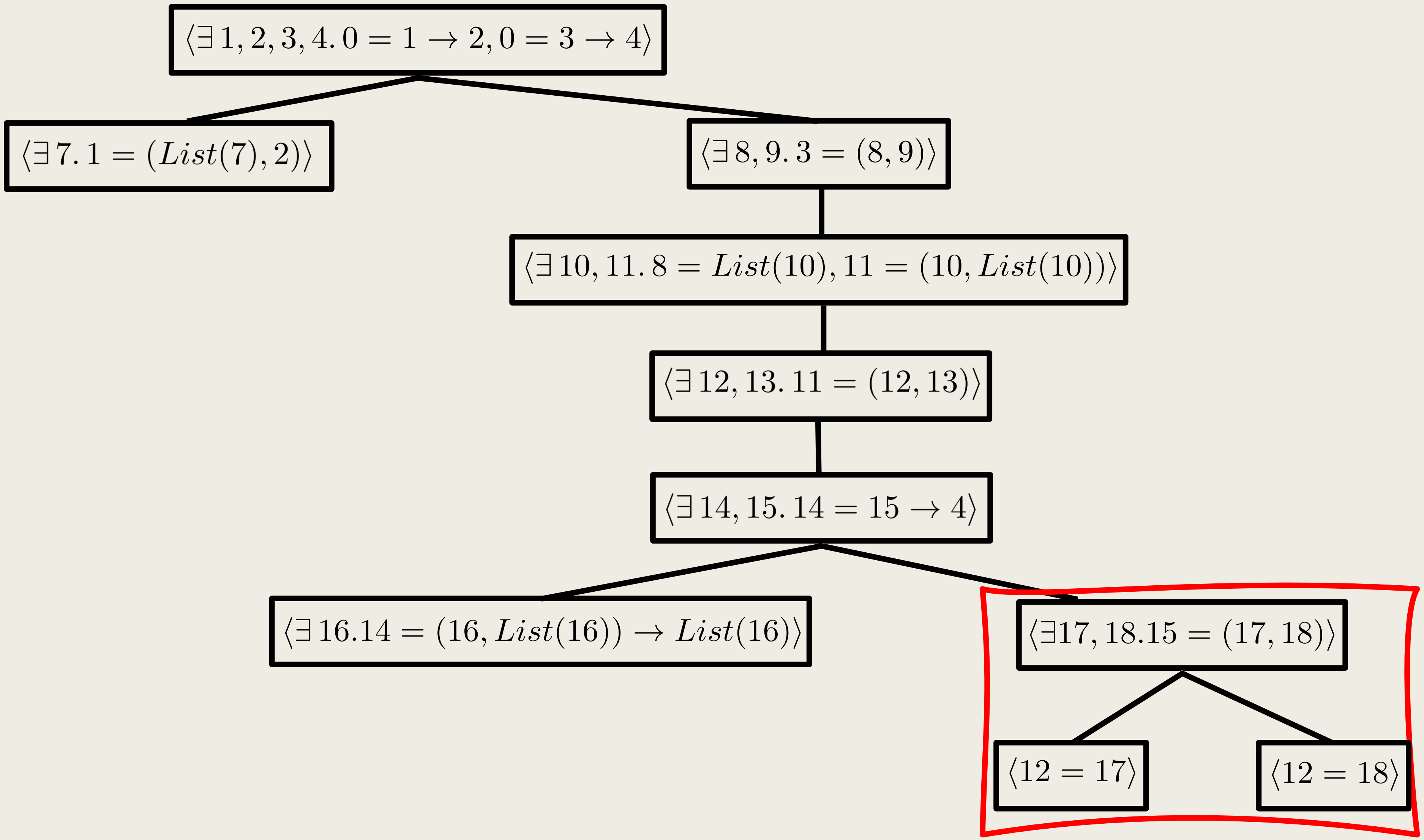
# Solving Equations




$\langle \exists\, 1, 2, 3, 4.\ 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
$\langle \exists\, 5, 6.\ 1 = (5, 6) \rangle$
$\langle \exists\, 8, 9.\ 3 = (8, 9) \rangle$
$\langle \exists\, 7.\ 5 = \mathit{List}(7) \rangle$
$\langle \exists\, 10, 11.\ 8 = \mathit{List}(10), 11 = (10, \mathit{List}(10)) \rangle$
$\langle 2 = 6 \rangle$
$\langle \exists\, 12, 13.\ 11 = (12, 13) \rangle$
$\langle \exists\, 14, 15.\ 14 = 15 \to 4 \rangle$
$\langle \exists\, 16.14 = (16, \mathit{List}(16)) \to \mathit{List}(16) \rangle$
$\langle \exists 17, 18.15 = (17, 18) \rangle$
$\langle 12 = 17 \rangle$
$\langle 12 = 18 \rangle$


$\langle \exists\, 1, 2, 3, 4.\, 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
$\langle \exists\, 5, 6.\, 1 = (5, 6) \rangle$
$\langle \exists\, 7.\, 5 = List(7), 2 = 6 \rangle$
$\langle \exists\, 8, 9.\, 3 = (8, 9) \rangle$
$\langle \exists\, 10, 11.\, 8 = List(10), 11 = (10, List(10)) \rangle$
$\langle \exists\, 12, 13.\, 11 = (12, 13) \rangle$
$\langle \exists\, 14, 15.\, 14 = 15 \to 4 \rangle$
$\langle \exists\, 16. 14 = (16, List(16)) \to List(16) \rangle$
$\langle \exists 17, 18. 15 = (17, 18) \rangle$
$\langle 12 = 17 \rangle$
$\langle 12 = 18 \rangle$


$\langle \exists\, 1,2,3,4.\, 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
$\langle 1 = (List(7), 2) \rangle$
$\langle \exists\, 5, 6.\, 1 = (5,6) \rangle$
$\langle \exists\, 8, 9.\, 3 = (8,9) \rangle$
$\langle \exists\, 7.\, 5 = List(7), 2 = 6 \rangle$
$\langle \exists\, 10, 11.\, 8 = List(10), 11 = (10, List(10)) \rangle$
$\langle \exists\, 12, 13.\, 11 = (12, 13) \rangle$
$\langle \exists\, 14, 15.\, 14 = 15 \to 4 \rangle$
$\langle \exists\, 16. 14 = (16, List(16)) \to List(16) \rangle$
$\langle \exists 17, 18. 15 = (17, 18) \rangle$
$\langle 12 = 17 \rangle$
$\langle 12 = 18 \rangle$


$\langle \exists\, 1, 2, 3, 4.\, 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
$\langle \exists\, 7.\, 1 = (List(7), 2) \rangle$
$\langle \exists\, 8, 9.\, 3 = (8, 9) \rangle$
$\langle \exists\, 10, 11.\, 8 = List(10), 11 = (10, List(10)) \rangle$
$\langle \exists\, 12, 13.\, 11 = (12, 13) \rangle$
$\langle \exists\, 14, 15.\, 14 = 15 \to 4 \rangle$
$\langle \exists\, 16.14 = (16, List(16)) \to List(16) \rangle$
$\langle \exists 17, 18.15 = (17, 18) \rangle$
$\langle 12 = 17 \rangle$
$\langle 12 = 18 \rangle$

$$\langle \exists\, 1, 2, 3, 4.\ 0 = 1 \to 2, 0 = 3 \to 4 \rangle$$

$$\langle \exists\, 7.\ 1 = (List(7), 2) \rangle$$

$$\langle \exists\, 8, 9.\ 3 = (8, 9) \rangle$$

$$\langle \exists\, 10, 11.\ 8 = List(10), 11 = (10, List(10)) \rangle$$

$$\langle \exists\, 12, 13.\ 11 = (12, 13) \rangle$$

$$\langle \exists\, 14, 15.\ 14 = 15 \to 4 \rangle$$

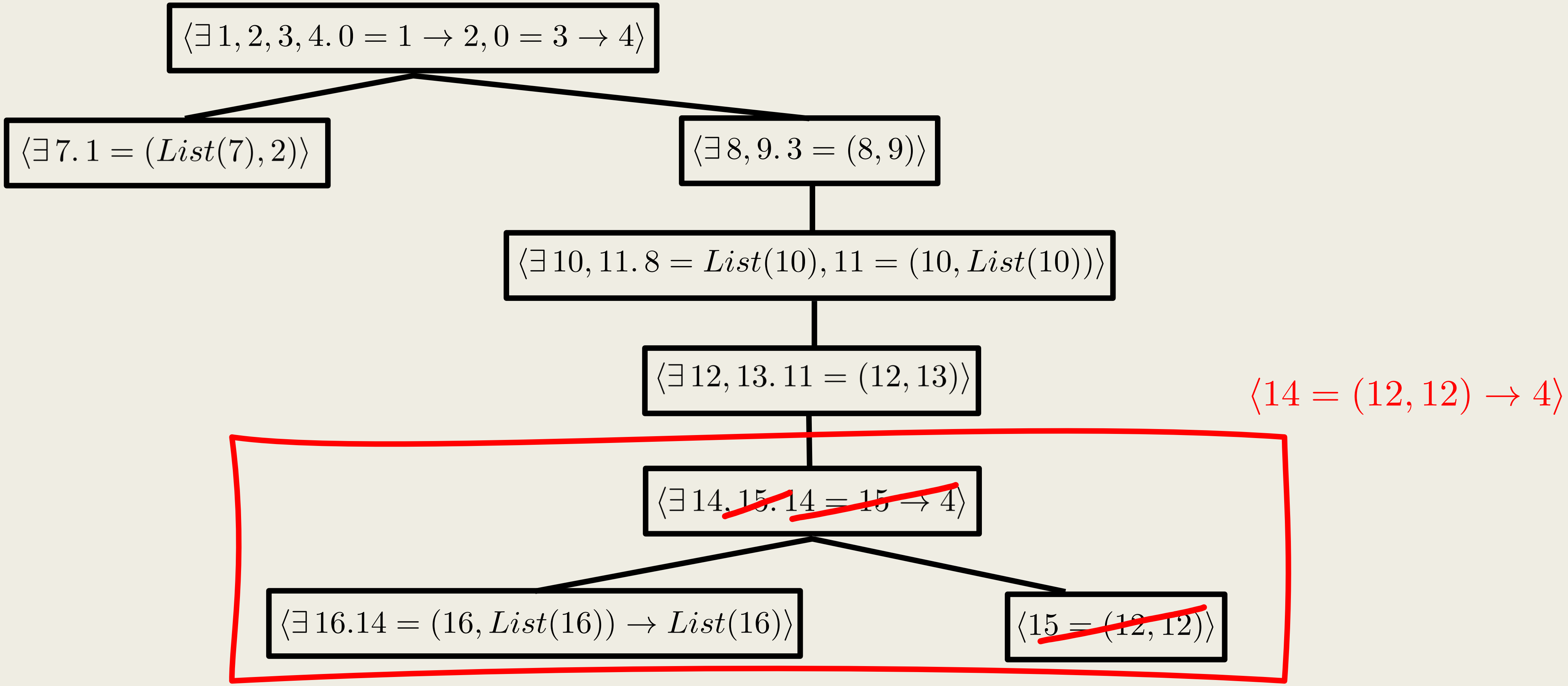
$$\langle \exists\, 16. 14 = (16, List(16)) \to List(16) \rangle$$

$$\langle 15 = (12, 12) \rangle$$

$$\langle \exists \cancel{17, 18.} 15 \cancel{= (17, 18)} \rangle$$

$$\cancel{\langle 12 = 17 \rangle} \qquad \cancel{\langle 12 = 18 \rangle}$$

- $\langle \exists\, 1, 2, 3, 4.\, 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
  - $\langle \exists\, 7.\, 1 = (List(7), 2) \rangle$
  - $\langle \exists\, 8, 9.\, 3 = (8, 9) \rangle$
    - $\langle \exists\, 10, 11.\, 8 = List(10), 11 = (10, List(10)) \rangle$
      - $\langle \exists\, 12, 13.\, 11 = (12, 13) \rangle$
        - $\langle \exists\, 14, 15.\, 14 = 15 \to 4 \rangle$
          - $\langle \exists\, 16.14 = (16, List(16)) \to List(16) \rangle$
          - $\langle 15 = (12, 12) \rangle$


$\langle \exists\, 1, 2, 3, 4.\ 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
$\langle \exists\, 7.\ 1 = (List(7), 2) \rangle$
$\langle \exists\, 8, 9.\ 3 = (8, 9) \rangle$
$\langle \exists\, 10, 11.\ 8 = List(10), 11 = (10, List(10)) \rangle$
$\langle \exists\, 12, 13.\ 11 = (12, 13) \rangle$
$\langle 14 = (12, 12) \to 4 \rangle$
$\langle \exists\, 14, \cancel{15}.\ \cancel{14 = 15 \to 4} \rangle$
$\langle \exists\, 16. 14 = (16, List(16)) \to List(16) \rangle$
$\langle \cancel{15 = (12, 12)} \rangle$

$$\langle \exists\, 1,2,3,4.\ 0 = 1 \to 2,\ 0 = 3 \to 4 \rangle$$

$$\langle \exists\, 7.\ 1 = (\mathit{List}(7), 2) \rangle$$

$$\langle \exists\, 8,9.\ 3 = (8,9) \rangle$$

$$\langle \exists\, 10,11.\ 8 = \mathit{List}(10),\ 11 = (10, \mathit{List}(10)) \rangle$$

$$\langle \exists\, 12,13.\ 11 = (12,13) \rangle$$

$$\boxed{\langle \exists\, 14,16.\ 14 = (12,12) \to 4,\ 14 = (16, \mathit{List}(16)) \to \mathit{List}(16) \rangle}$$

$$\langle \exists\, 1, 2, 3, 4.\, 0 = 1 \to 2, 0 = 3 \to 4 \rangle$$

$$\langle \exists\, 7.\, 1 = (List(7), 2) \rangle$$

$$\langle \exists\, 8, 9.\, 3 = (8, 9) \rangle$$

$$\langle \exists\, 10, 11.\, 8 = List(10), 11 = (10, List(10)) \rangle$$

$$\langle \exists\, 12, 13.\, 11 = (12, 13) \rangle$$

$$\langle \exists\, \cancel{14,} 16.\, \cancel{14 = (12, 12)} \to 4, \cancel{14 = (16, List(16)) \to List(16)} \rangle$$

$$\langle (12, 12) \to 4 = (16, List(16)) \to List(16) \rangle$$

matching

$$\langle (12, 12) = (16, List(16)) \rangle$$
$$\langle 4 = List(16) \rangle$$

matching

$$\langle 12 = 16 \rangle$$
$$\langle 12 = List(16) \rangle$$

- $\langle \exists\, 1, 2, 3, 4.\ 0 = 1 \to 2, 0 = 3 \to 4 \rangle$
  - $\langle \exists\, 7.\ 1 = (\mathit{List}(7), 2) \rangle$
  - $\langle \exists\, 8, 9.\ 3 = (8, 9) \rangle$
    - $\langle \exists\, 10, 11.\ 8 = \mathit{List}(10), 11 = (10, \mathit{List}(10)) \rangle$
      - $\langle \exists\, 12, 13.\ 11 = (12, 13) \rangle$
        - $\langle \exists\ 16.\ 12 = 16, 12 = \mathit{List}(16), 4 = \mathit{List}(16) \rangle$

$$\langle \exists\, 1, 2, 3, 4.\; 0 = 1 \to 2, 0 = 3 \to 4 \rangle$$

$$\langle \exists\, 7.\; 1 = (List(7), 2) \rangle \qquad \langle \exists\, 8, 9.\; 3 = (8, 9) \rangle$$

$$\langle \exists\, 10, 11.\; 8 = List(10), 11 = (10, List(10)) \rangle$$

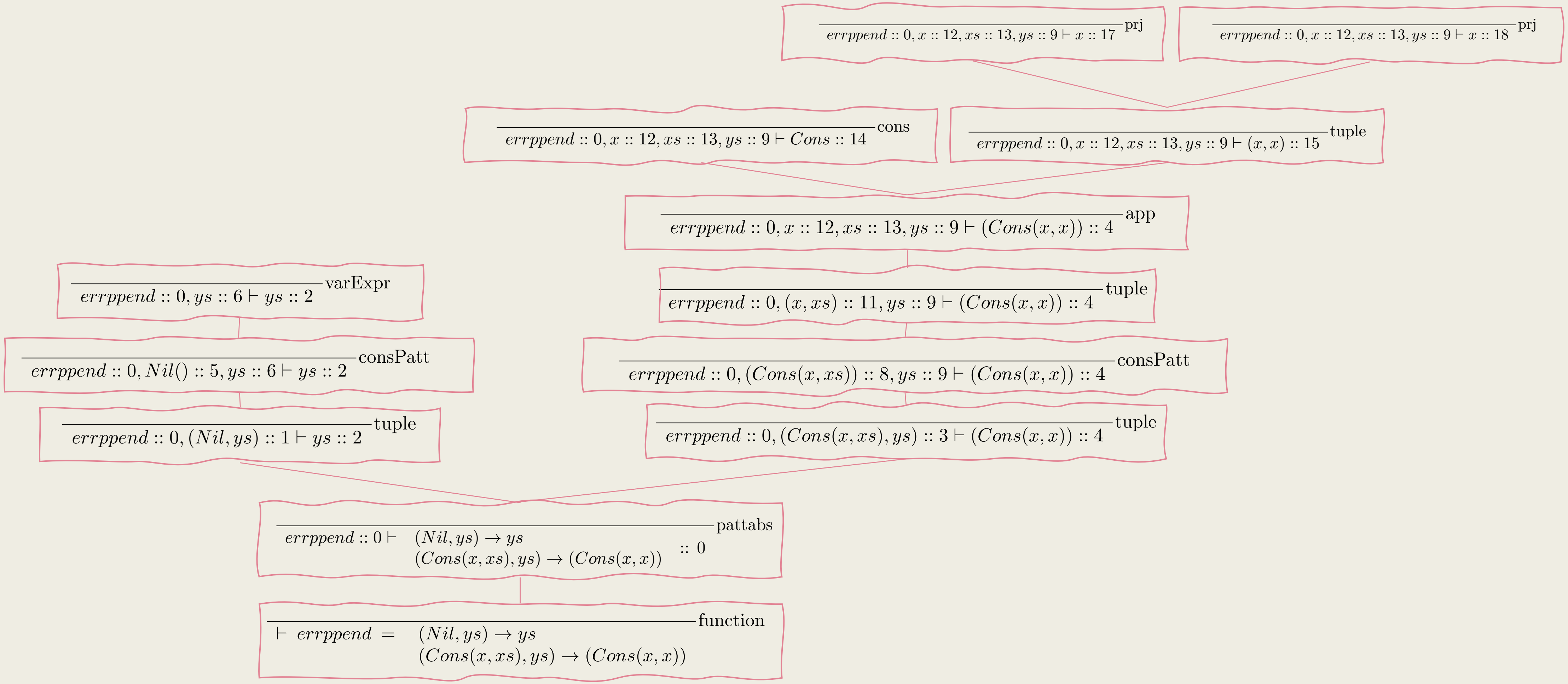
$$\langle \exists\, \cancel{12,}\, 13, 16.\; 11 = (12, 13), \cancel{12 = 16}, \cancel{12 = List(16)}, 4 = List(16) \rangle$$

$\langle 16 = List(16) \rangle$ Occurs Check Failure

The second step of type inference in this algorithm uses all the equations and we do not need all of them to find there is an error, also the location of the error is not tracked.

We thought of a new approach to address this problem.

# New Approach To Solve Equations



$$\frac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj}$$

$$\frac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt}$$

$$\frac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0}\,\text{pattabs}$$

$$\frac{}{\vdash errppend \ = \ \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array}}\,\text{function}$$

# Type Inference

$$\frac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj}$$

$$\frac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt}$$

$$\frac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0}\,\text{pattabs}$$

$$\frac{}{\vdash\ errppend\ =\ \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array}}\,\text{function}$$

# Equations

# Type Inference

1

$$\frac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\text{prj}$$

$$E_0 = \langle 2 = 6 \rangle$$

$$\frac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\text{consPatt}$$

$$\frac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0}\text{pattabs}$$

$$\frac{}{\vdash\ errppend\ =\ \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array}}\text{function}$$

# Equations

$$\boxed{\langle 2 = 6 \rangle}$$

# Type Inference

1
$$\dfrac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\,\text{varExpr}$$
$$E_0 = \langle 2 = 6 \rangle$$

2
$$\dfrac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt}$$
$$E_1 = \langle \exists\, 7.5 = List(7), E_0 \rangle$$

$$\dfrac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple}$$

$$\dfrac{}{errppend :: 0 \vdash \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array} :: 0}\,\text{pattabs}$$

$$\dfrac{}{\vdash errppend = \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array}}\,\text{function}$$

# Equations

$$\boxed{\langle \exists\, 7.5 = List(7) \rangle}$$
$$\boxed{\langle 2 = 6 \rangle}$$

# Type Inference

1 $$\dfrac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj}$$
$$E_0 = \langle 2 = 6 \rangle$$

2 $$\dfrac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt}$$
$$E_1 = \langle \exists\, 7.5 = List(7), E_0 \rangle$$

3 $$\dfrac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple}$$
$$E_3 = \langle \exists\, 5, 6.1 = (5,6), E_2 \rangle$$

$$\dfrac{}{errppend :: 0 \vdash \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array} :: 0}\,\text{pattabs}$$

$$\dfrac{}{\vdash\ errppend\ = \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array}}\,\text{function}$$

# Equations

$$\boxed{\langle \exists\, 5, 6.\, 1 = (5,6) \rangle}$$
$$\boxed{\langle \exists\, 7.\, 5 = List(7) \rangle}$$
$$\boxed{\langle 2 = 6 \rangle}$$

$\rightarrow$

$$\boxed{\langle \exists\, \cancel{5, 6.\, 1 = (5,6)} \rangle}$$
$$\boxed{\langle \exists\, 7.\cancel{5 = List(7), 2 = 6} \rangle}$$

$$\langle 1 = (List(7), 2) \rangle$$

$\rightarrow$

$$\boxed{\langle \exists\, 7.\, 1 = (List(7), 2) \rangle}$$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17}\ \text{prj} \qquad \frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18}\ \text{prj}$$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14}\ \text{cons} \qquad \frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15}\ \text{tuple}$$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x, x)) :: 4}\ \text{app}$$

$$\frac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\ \text{prj} \qquad \frac{}{errppend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x, x)) :: 4}\ \text{tuple}$$

$$\frac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\ \text{consPatt} \qquad \frac{}{errppend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x, x)) :: 4}\ \text{consPatt}$$

$$\frac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\ \text{tuple} \qquad \frac{}{errppend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x, x)) :: 4}\ \text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0}\ \text{pattabs}$$

$$\frac{}{\vdash errppend = \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array}}\ \text{function}$$

# Type Inference

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17}\,\text{prj}$$

$$\frac{}{\begin{array}{c}errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18 \\ E_4 = \langle 12 = 18 \rangle\end{array}}\,\text{prj}$$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14}\,\text{cons}$$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15}\,\text{tuple}$$

$$\boxed{\langle \exists\, 7.\, 1 = (List(7), 2) \rangle}$$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{app}$$

1

$$\frac{}{\begin{array}{c}errppend :: 0, ys :: 6 \vdash ys :: 2 \\ E_0 = \langle 2 = 6 \rangle\end{array}}\,\text{prj}$$

$$\frac{}{errppend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

2

$$\frac{}{\begin{array}{c}errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2 \\ E_1 = \langle \exists\, 7.5 = List(7), E_0 \rangle\end{array}}\,\text{consPatt}$$

$$\frac{}{errppend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{consPatt}$$

3

$$\frac{}{\begin{array}{c}errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2 \\ E_3 = \langle \exists\, 5, 6.1 = (5, 6), E_2 \rangle\end{array}}\,\text{tuple}$$

$$\frac{}{errppend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array} :: 0}\,\text{pattabs}$$

$$\frac{}{\vdash\ errppend\ =\ \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array}}\,\text{function}$$

4

# Equations

$$\boxed{\langle 12 = 18 \rangle}$$

# Type Inference

5

$\boxed{\langle 12 = 18 \rangle}$

4

$$\frac{}{\mathit{errppend} :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17}\,\text{prj} \qquad E_5 = \langle 12 = 17 \rangle$$

$$\frac{}{\mathit{errppend} :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18}\,\text{prj} \qquad E_4 = \langle 12 = 18 \rangle$$

$$\frac{}{\mathit{errppend} :: 0, x :: 12, xs :: 13, ys :: 9 \vdash \mathit{Cons} :: 14}\,\text{cons}$$

$$\frac{}{\mathit{errppend} :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15}\,\text{tuple}$$

$\boxed{\langle \exists\, 7.\, 1 = (\mathit{List}(7), 2) \rangle}$

$$\frac{}{\mathit{errppend} :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (\mathit{Cons}(x, x)) :: 4}\,\text{app}$$

1

$$\frac{}{\mathit{errppend} :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj}$$
$$E_0 = \langle 2 = 6 \rangle$$

$$\frac{}{\mathit{errppend} :: 0, (x, xs) :: 11, ys :: 9 \vdash (\mathit{Cons}(x, x)) :: 4}\,\text{tuple}$$

2

$$\frac{}{\mathit{errppend} :: 0, \mathit{Nil}() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt}$$
$$E_1 = \langle \exists\, 7.5 = \mathit{List}(7), E_0 \rangle$$

$$\frac{}{\mathit{errppend} :: 0, (\mathit{Cons}(x, xs)) :: 8, ys :: 9 \vdash (\mathit{Cons}(x, x)) :: 4}\,\text{consPatt}$$

3

$$\frac{}{\mathit{errppend} :: 0, (\mathit{Nil}, ys) :: 1 \vdash ys :: 2}\,\text{tuple}$$
$$E_3 = \langle \exists\, 5, 6.1 = (5, 6), E_2 \rangle$$

$$\frac{}{\mathit{errppend} :: 0, (\mathit{Cons}(x, xs), ys) :: 3 \vdash (\mathit{Cons}(x, x)) :: 4}\,\text{tuple}$$

$$\frac{}{\mathit{errppend} :: 0 \vdash \begin{array}{l} (\mathit{Nil}, ys) \to ys \\ (\mathit{Cons}(x, xs), ys) \to (\mathit{Cons}(x, x)) \end{array} :: 0}\,\text{pattexps}$$

$$\frac{}{\vdash\ \mathit{errppend}\ =\ \begin{array}{l} (\mathit{Nil}, ys) \to ys \\ (\mathit{Cons}(x, xs), ys) \to (\mathit{Cons}(x, x)) \end{array}}\,\text{function}$$

# Equations

$\boxed{\langle 12 = 17 \rangle}$

# Type Inference

5

$$\dfrac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17 \quad E_5 = \langle 12 = 17 \rangle}\,\text{prj}$$

4

$$\dfrac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18 \quad E_4 = \langle 12 = 18 \rangle}\,\text{prj}$$

$$\boxed{\langle \exists\, 7.\, 1 = (List(7), 2) \rangle}$$

$$\dfrac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14}\,\text{cons}$$

6

$$\dfrac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15 \quad E_6 = \langle \exists\, 17, 18.15 = (17, 18), E_4, E_5 \rangle}\,\text{tuple}$$

$$\dfrac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{app}$$

1

$$\dfrac{}{errppend :: 0, ys :: 6 \vdash ys :: 2 \quad E_0 = \langle 2 = 6 \rangle}\,\text{prj}$$

$$\dfrac{}{errppend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

2

$$\dfrac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2 \quad E_1 = \langle \exists\, 7.5 = List(7), E_0 \rangle}\,\text{consPatt}$$

$$\dfrac{}{errppend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{consPatt}$$

3

$$\dfrac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2 \quad E_3 = \langle \exists\, 5, 6.1 = (5, 6), E_2 \rangle}\,\text{tuple}$$

$$\dfrac{}{errppend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

$$\dfrac{}{errppend :: 0 \vdash \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0}\,\text{pattabs}$$

$$\dfrac{}{\vdash errppend = \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array}}\,\text{function}$$

# Equations

$$\boxed{\langle \exists \cancel{17, 18}.15 = \cancel{(17, 18)} \rangle}$$

$$\boxed{\cancel{\langle 12 = 17 \rangle}} \qquad \boxed{\cancel{\langle 12 = 18 \rangle}}$$

$$\rightarrow \qquad \boxed{\langle 15 = (12, 12) \rangle}$$

# Type Inference

$\langle 15 = (12,12)\rangle$

5
$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17}\,\text{prj} \quad E_5 = \langle 12 = 17\rangle$$

4
$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18}\,\text{prj} \quad E_4 = \langle 12 = 18\rangle$$

7
$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14}\,\text{cons} \quad E_7 = \langle \exists 16.14 = (16, List(16)) \to List(16)\rangle$$

6
$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x,x) :: 15}\,\text{tuple} \quad E_6 = \langle \exists\, 17, 18.15 = (17,18), E_4, E_5\rangle$$

$\langle \exists\, 7.\, 1 = (List(7), 2)\rangle$

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x,x)) :: 4}\,\text{app}$$

1
$$\frac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj} \quad E_0 = \langle 2 = 6\rangle$$

$$\frac{}{errppend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x,x)) :: 4}\,\text{tuple}$$

2
$$\frac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt} \quad E_1 = \langle \exists\, 7.5 = List(7), E_0\rangle$$

$$\frac{}{errppend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x,x)) :: 4}\,\text{consPatt}$$

3
$$\frac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple} \quad E_3 = \langle \exists\, 5, 6.1 = (5,6), E_2\rangle$$

$$\frac{}{errppend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x,x)) :: 4}\,\text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x,xs), ys) \to (Cons(x,x))\end{array} :: 0}\,\text{pattabs}$$

$$\frac{}{\vdash errppend = \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x,xs), ys) \to (Cons(x,x))\end{array}}\,\text{function}$$

# Equations

$\langle \exists 16.14 = (16, List(16)) \to List(16)\rangle$

# Type Inference

$$\boxed{\langle \exists 7.\, 1 = (List(7), 2) \rangle}$$

1

$$\frac{}{errppend :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj} \qquad E_0 = \langle 2 = 6 \rangle$$

2

$$\frac{}{errppend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt} \qquad E_1 = \langle \exists 7.5 = List(7), E_0 \rangle$$

3

$$\frac{}{errppend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple} \qquad E_3 = \langle \exists 5, 6.1 = (5, 6), E_2 \rangle$$

5

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17}\,\text{prj} \qquad E_5 = \langle 12 = 17 \rangle$$

4

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18}\,\text{prj} \qquad E_4 = \langle 12 = 18 \rangle$$

7

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14}\,\text{cons} \qquad E_7 = \langle \exists 16.14 = (16, List(16)) \to List(16) \rangle$$

6

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15}\,\text{tuple} \qquad E_6 = \langle \exists 17, 18.15 = (17, 18), E_4, E_5 \rangle$$

8

$$\frac{}{errppend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{app} \qquad E_8 = \langle \exists 14, 15.14 = 15 \to 4.E_6, E_7 \rangle$$

$$\frac{}{errppend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

$$\frac{}{errppend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{consPatt}$$

$$\frac{}{errppend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

$$\frac{}{errppend :: 0 \vdash \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array} :: 0}\,\text{pattabs}$$

$$\frac{}{\vdash errppend = \begin{array}{l}(Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x))\end{array}}\,\text{function}$$

# Equations

$$\langle 14 = (12, 12) \to 4 \rangle$$

$$\boxed{\langle \exists 14, \cancel{15.\,14 = 15 \to 4} \rangle}$$

$$\boxed{\langle \exists 16.14 = (16, List(16)) \to List(16) \rangle} \qquad \boxed{\langle \cancel{15 = (12, 12)} \rangle}$$

$$\boxed{\langle \exists \cancel{14,} 16.\, \cancel{14 = (12, 12) \to 4, 14 = (16, List(16)) \to List(16)} \rangle}$$

$$\langle (12, 12) \to 4 = (16, List(16)) \to List(16) \rangle$$

matching

$$\langle (12, 12) = (16, List(16)) \rangle \quad \langle 4 = List(16) \rangle$$

matching

$$\langle 12 = 16 \rangle \quad \langle 12 = List(16) \rangle$$

$$\boxed{\langle \exists 16.\, 12 = 16, 12 = List(16), 4 = List(16) \rangle}$$

# Type Inference

$$\boxed{\langle \exists\, 7.\, 1 = (List(7), 2)\rangle}$$

5
$$\frac{}{errpend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 17}\,\text{prj} \qquad E_5 = \langle 12 = 17\rangle$$

4
$$\frac{}{errpend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash x :: 18}\,\text{prj} \qquad E_4 = \langle 12 = 18\rangle$$

7
$$\frac{}{errpend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash Cons :: 14}\,\text{cons} \qquad E_7 = \langle \exists 16.14 = (16, List(16)) \to List(16)\rangle$$

6
$$\frac{}{errpend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (x, x) :: 15}\,\text{tuple} \qquad E_6 = \langle \exists\, 17, 18.15 = (17, 18), E_4, E_5\rangle$$

8
$$\frac{}{errpend :: 0, x :: 12, xs :: 13, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{appliedExpr} \qquad E_8 = \langle \exists\, 14, 15.14 = 15 \to 4.E_6, E_7\rangle$$

9
$$\frac{}{errpend :: 0, (x, xs) :: 11, ys :: 9 \vdash (Cons(x)) :: 4}\,\text{tuple} \qquad E_7 = \langle \exists\, 12, 13.11 = (12, 13), E_{10}\rangle$$

$$\frac{}{errpend :: 0, (Cons(x, xs)) :: 8, ys :: 9 \vdash (Cons(x, x)) :: 4}\,\text{consPatt}$$

$$\frac{}{errpend :: 0, (Cons(x, xs), ys) :: 3 \vdash (Cons(x, x)) :: 4}\,\text{tuple}$$

1
$$\frac{}{errpend :: 0, ys :: 6 \vdash ys :: 2}\,\text{prj} \qquad E_0 = \langle 2 = 6\rangle$$

2
$$\frac{}{errpend :: 0, Nil() :: 5, ys :: 6 \vdash ys :: 2}\,\text{consPatt} \qquad E_1 = \langle \exists\, 7.5 = List(7), E_0\rangle$$

3
$$\frac{}{errpend :: 0, (Nil, ys) :: 1 \vdash ys :: 2}\,\text{tuple} \qquad E_3 = \langle \exists\, 5, 6.1 = (5, 6), E_2\rangle$$

$$\frac{}{errpend :: 0 \vdash \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array} :: 0}\,\text{pattabs}$$

$$\frac{}{\vdash errpend = \begin{array}{l} (Nil, ys) \to ys \\ (Cons(x, xs), ys) \to (Cons(x, x)) \end{array}}\,\text{function}$$

# Equations

$$\boxed{\langle \exists\, \cancel{12,}\, 13, 16.\, 11 = (12, 13), \cancel{12 = 16}, \cancel{12 = List(16)}, 4 = List(16)\rangle}$$

$\langle 16 = List(16)\rangle$ Occurs Check Failure

# Fold for Inductive Data Type

fold construct

```
append = xs, ys -> fold xs of
                     Nil : -> ys
                     Cons : (b, acc) -> (b:acc)
```

# CaMPL Fold Rules

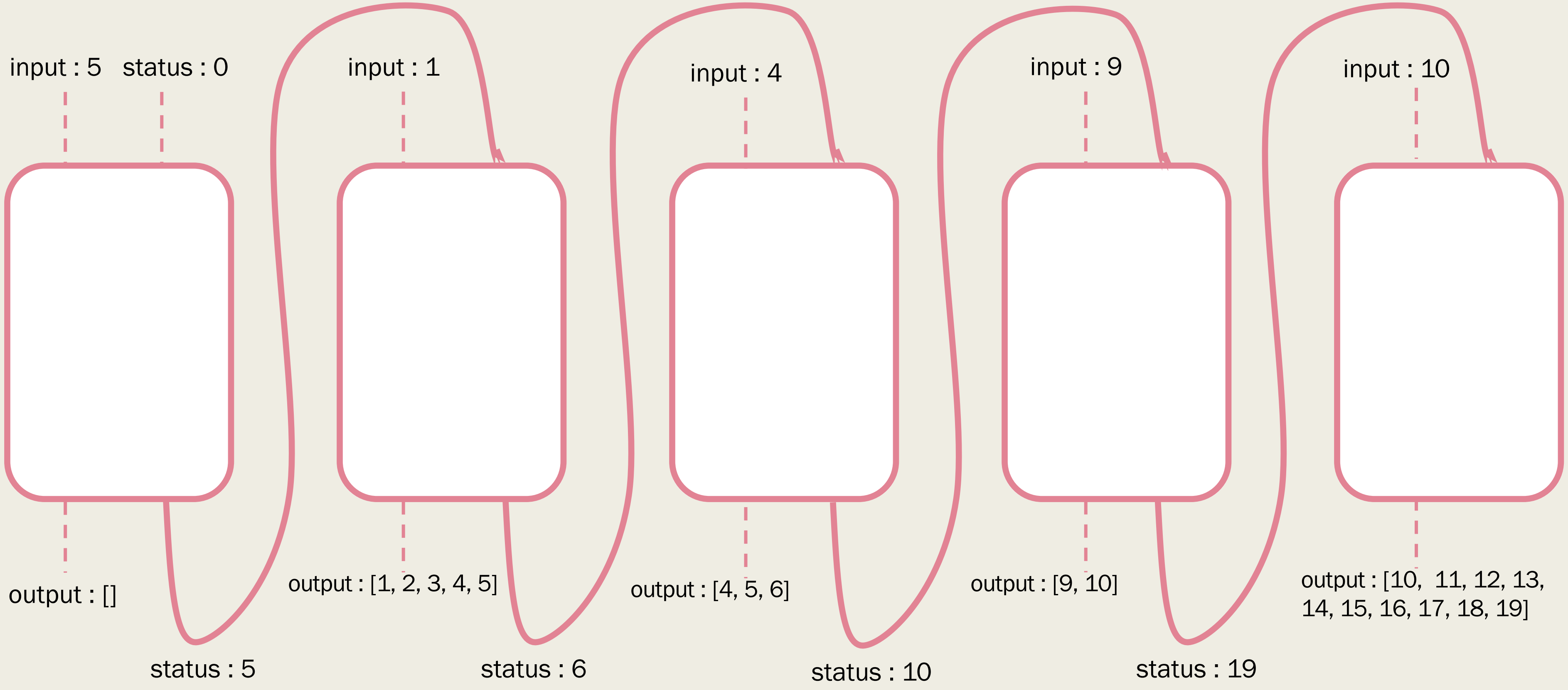
$$\begin{array}{llll} data & List(A) & \rightarrow & C = \\ & Nil :: & \rightarrow & C \\ & Cons :: A, C & \rightarrow & C \end{array} \xrightarrow{[5/A,\ 0/C]} \begin{array}{lll} Nil :: & \rightarrow & 0 \\ Cons :: 5, 0 & \rightarrow & 0 \end{array}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash xs :: 1 \\ \langle E_1 \rangle\end{array} \qquad \begin{array}{c}\Gamma \vdash ys :: 2 \\ \langle E_2 \rangle\end{array} \qquad \begin{array}{c}\Gamma, (b, acc) :: 3 \vdash (b : acc) :: 4 \\ \langle E_3 \rangle\end{array}}{\begin{array}{c}\Gamma \vdash \begin{array}{l} fold \quad xs \quad of \\ \quad Nil : \rightarrow ys \\ \quad Cons : (b, acc) \rightarrow (b : acc) \end{array} :: 0 \\ \langle \exists\, 1, 2, 3, 4, 5.\ 1 = List(5), 2 = 0, 3 = (5, 0), 4 = 0, E_1, E_2, E_3 \rangle\end{array}}\ \text{fold}$$

# CaMPL CoData Constructs

```
codata C -> Mealy (A, B) =
    Step :: A, C -> (B, C)
```

# Mealy Machine



# CaMPL CoData Constructs

```
codata C -> Mealy(A, B) =
    Step :: A, C -> (B, C)


fun unfoldmealy = f, c -> (Step := a -> case (f(a,c)) of
                             (b, cp) -> (b, unfoldmealy(f, cp)))
```

recordExpr

# CaMPL CoData Type Rules

$$\begin{array}{lll} codata & C \to Mealy(A,B) & = \\ & Step : A, C \to (B, C) & \end{array} \xrightarrow{[3/A,\ 4/B,\ ,Mealy(3,4)/C]} Step : 3, Mealy(3,4) \to (4, Mealy(3,4))$$

$$\dfrac{\begin{array}{l} a :: 1 \vdash\ case(f(a,c))\ of \\ \qquad (b, cp) \to\ (b, unfoldmealy(f, cp)) \\ \qquad\qquad \langle E_1 \rangle \end{array} \quad :: \ 2}{\begin{array}{l} \vdash\ (Step := a \to\ case(f(a,c))\ of \\ \qquad (b, cp) \to\ (b, unfoldmealy(f, cp)) \\ \qquad E_0 = \langle \exists\, 1, 2, 3, 4.1 = 3, 2 = (4, Mealy(3,4)), 0 = Mealy(3,4), E_1 \rangle \end{array} \quad :: \ 0}\ \text{recordExpr}$$

# CaMPL Constructs

There are more constructs in CaMPL such as:

1. Defn and Where Constructs
2. Concurrent constructs such as : run, close, put, get, halt, hcase, hput, split, fork, Id, Neg, Plug, Race, Drive, …

By writing the rules for all of these constructs, CaMPL will have a powerful type inference system.

# Conclusion

- Type judgments can be written for every construct in CaMPL. If we write these type judgments systematically, we will be able to use them in the new constructs that we face in the language as well.

- CaMPL has both sequential and concurrent constructs. Type judgements can be written for all of these constructs.

- Our approach is to combine the two steps of the type inference algorithm into one step, so that errors can be detected as soon as possible and the location of the type errors will be more accurate.

# References


1. J. R. B. Cockett and Craig Pastro. The Logic of Message Passing. Science of Computer Programming, 74(8):498–533, 2009.

2. Reginald Lybbert. Progress for the Message Passing Logic. Undergraduate thesis, University of Calgary, April 2018. Provided by the author.

3. Prashant Kumar. Implementation of Message Passing Language. 2018. doi:10.11575/PRISM/10182. url: https://prism.ucalgary.ca/ handle/1880/106402.

4. Jared Pon. Implementation Status of CMPL. Undergraduate thesis Interim Report, University of Calgary, December 2021. Provided by the author.

5. J. R. B. Cockett. Basic Programming for Computable Functions: BPCF (The Typed Lambda Calculus with Fixed Points) , April 9, 2024. Provided by the author.

Thank You ☺